Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2014

Name: SOLUTIONS

Instructor: Galvin/Diaz

Practice exam 3

April 17, 2014

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

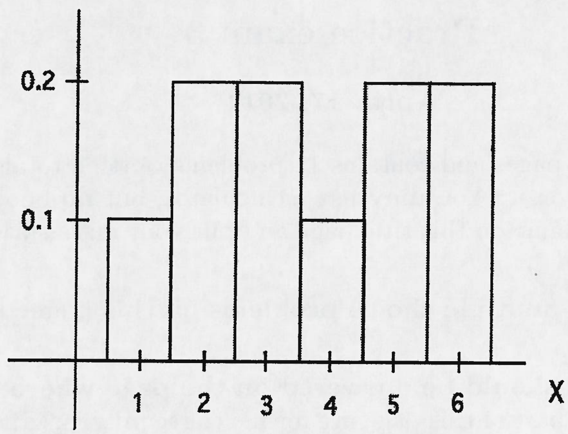
May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) The picture shows the histogram for a probability distribution X. What is the expected value of X?



(a) 3.5 (b) 3.2 (c) 3.7 (d) 4 (e) 3

$E(X) = 1(.1) + 2(.2) + 3(.2) + 4(.1) + 5(.2) + 6(.2)$

$= 3.7$

2. (5 pts.) According to Forbes, the University of Notre Dame (ND) is ranked 24 in the list of America's Top Colleges. There are 650 colleges in the list. In which percentile is ND?

(a) 95% (b) 96% (c) 5%

(d) 4% (e) 24%

~~650 - 24~~ 23 schools ranked higher than ND,

So $650 - 23 = 627$ ranked same or worse

627 is 96.46%; so ND is in 96th percentile

3. (5 pts.) What is the third quartile of the following scores?

6, 3, 9, 10, 10, 9, 7

(a) 8 (b) 9

(c) 7 (d) 9.5

(e) 10

3, 6, 7, 9, 9, 10, 10

median

median of numbers above median

4. (5 pts.) A multiple choice exam has 10 questions, each with five options (labelled (a), (b), (c), (d) and (e)). I haven't studied for the exam, so I take a random guess (all options equally likely) at each question. What is the probability that I get exactly 3 questions right?

(a) $(.8)^{10} + C(10,1)(.8)^9(.2) + C(10,2)(.8)^8(.2)^2$ (b) $1 - ((.8)^{10} + C(10,1)(.8)^9(.2) + C(10,2)(.8)^8(.2)^2)$

(c) $1 - (.8)^{10}$ (d) $C(10,3)(.8)^7(.2)^3$

(e) $1 - C(10,3)(.8)^7(.2)^3$

5. (5 pts.) Find the variance of the random variable defined by the following table

X	$P(X)$
10	0.4
30	0.2
50	0.3
80	0.1

(a) 521 (b) 33 (c) 493

(d) 0 (e) 4877

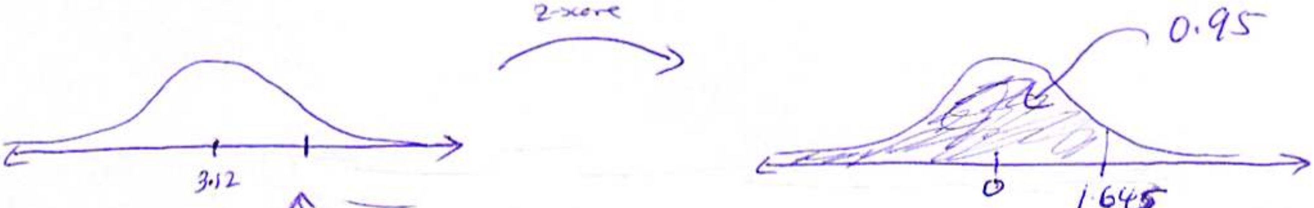
$$E(X) = 10(0.4) + 30(0.2) + 50(0.3) + 80(0.1)$$
$$= 33$$

$$\text{variance} = (10-33)^2(0.4) + (30-33)^2(0.2)$$
$$+ (50-33)^2(0.3) + (80-33)^2(0.1)$$

$$= 521.$$

6. (5 pts.) The College of Science assesses its professors' teaching using a "composite number" that is (somewhat mysteriously) produced from end-of-semester CIF's. The mean score for professors is 3.12, with standard deviation .21. What "composite number" do I need to have, to be better than 95% of all the professors in the College of Science? (Round your answer to two decimal places.)

(a) 3.52 (b) 3.20 (c) 3.36 (d) 3.47 (e) 3.33



From the table, we have that the area under the curve to the left of $z = 1.645$ is appx 0.95.

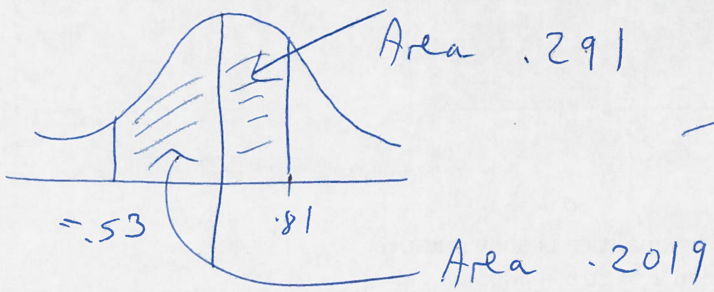
Now notice that $1.645 = \dfrac{\text{score} - 3.12}{0.21} \Rightarrow \boxed{\text{score} = 3.47}$

7. (5 pts.) One year the GRE test had a mean of $\mu = 459$ and a standard deviation $\sigma = 112$. The scores formed a normal distribution. Find the probability that a score lies between 400 and 550.

(a) 32% (b) 68% (c) 49% (d) 51% (e) 9%

z-score of 400 is $\frac{400-459}{112} = -.53$

z-score of 550 is $\frac{550-459}{112} = .81$



Total area .4929

8. (5 pts.) What are the coordinates of the point of intersection of the two given lines?

$$\begin{aligned} 5x - y &= 2 \\ -8x + 4y &= 4 \end{aligned}$$

(a) $x = 3, y = 13$ (b) $x = 2, y = 8$

(c) $x = 1/9, y = 13/9$ (d) $x = 2, y = 10$

(e) $x = 1, y = 3$

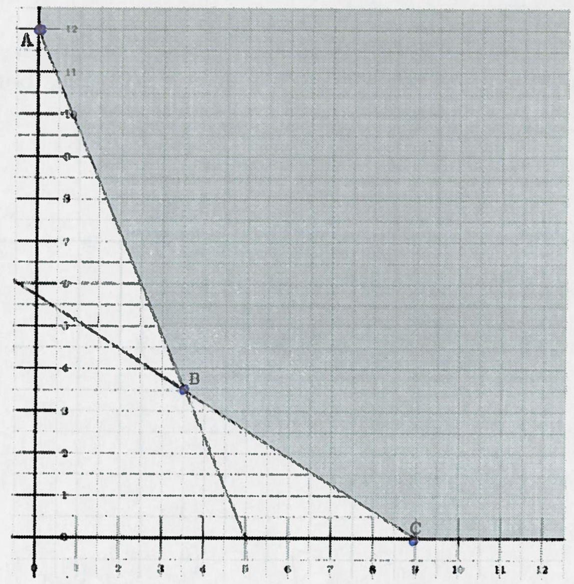
$5x - y = 2$ so $y = 5x - 2$

so $-8x + 4(5x-2) = 4$

so $12x = 12$

so $x = 1$

so $y = 3$



9. (5 pts.) The feasible region of a certain system of inequalities is shown above. What is the minimum value of the objective function $z = 20x + 30y$?

(a) 175 (b) 100

(c) 180 (d) 360

(e) 0

Corner point $x = 3.5, y = 3.5$, objective 175

Smallest corner point

Corner point $x=9, y=0 \to$ objective 180

Corner point $x=0, y=12 \to$ objective 360

10. (5 pts.) Which of the pair of values for x and y given below is in the feasible set for the given inequalities?

$$\begin{aligned} x + 2y &\geq 7 \\ 2x - y &\geq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(a) $x = 0$, $y = 4$

(b) $x = 3$, $y = -2$

(c) $x = 2$, $y = 1$

(d) $x = 7$, $y = 1$ ← Only this satisfies all 4 inequalities; each of the rest fails at least one.

(e) $x = 1$, $y = 3$

Partial Credit

You must show **all of your work** on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) The scores of top 10 finalists for the Uneven Parallel Bars at the USA Gymnastics Collegiate Nationals Event Finals 2014 are as follow:

9.85, 9.80, 9.82, 9.75, 9.70, 9.70, 9.85, 9.85, 9.80 ,9.82.

(a) Compute the mean, median and mode of the scores.

Mean $= \frac{\text{Sum}}{10} = 9.794$

Median $= 9.81$, the average of 9.8 and 9.82 [the numbers in 5^{th} and 6^{th} position when written in increasing order]

Mode $= 9.85$ (occurs 3 times)

(b) Compute the variance of the scores.

Sum of $\frac{(\text{numbers} - 9.794)^2}{10} = .00304$

Variance

(c) Compute the standard deviation of the scores.

Std dev $= \sqrt{\text{Variance}} = .05517$

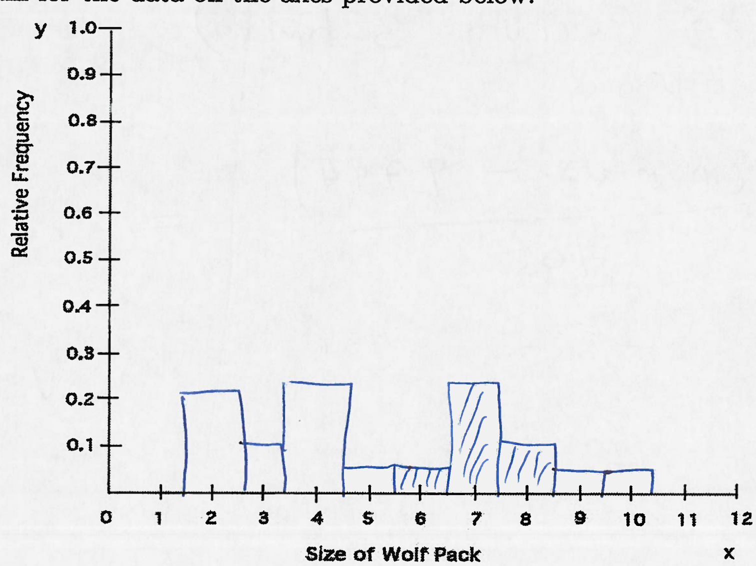
12. (10 pts.) Zoologists studying wolfpacks close to the arctic circle recorded the sizes of the packs that they found, as follows:

$$6,\ 10,\ 7,\ 5,\ 7,\ 7,\ 2,\ 4,\ 3,\ 2,\ 2,\ 3,\ 9,\ 4,\ 4,\ 2,\ 8,\ 7,\ 8,\ 4.$$

(a) Organize the data in a relative frequency table.

Outcome	Frequency	Rel. Freq.
2	4	4/20 = .2
3	2	2/20 = .1
4	4	4/20 = .2
5	1	1/20 = .05
6	1	1/20 = .05
7	4	4/20 = .2
8	2	2/20 = .1
9	1	1/20 = .05
10	1	1/20 = .05
Totals	20	

(b) Draw a histogram for the data on the axes provided below:



(c) Assuming that the samples observed represent the sizes of wolf packs accurately, use the data to estimate the probability that a randomly selected pack has size between 6 and 8, inclusive.

$.05 + .2 + .1 = .35$

13. (10 pts.) A company has a machine that produces screws. The probability that the machine produces a defective screw is 10%. On a certain day, the machine produced 200 screws. Let X be the number of defective screws.

(a) What is the probability that exactly 3 of them are defective, this is, what is $P(X = 3)$?

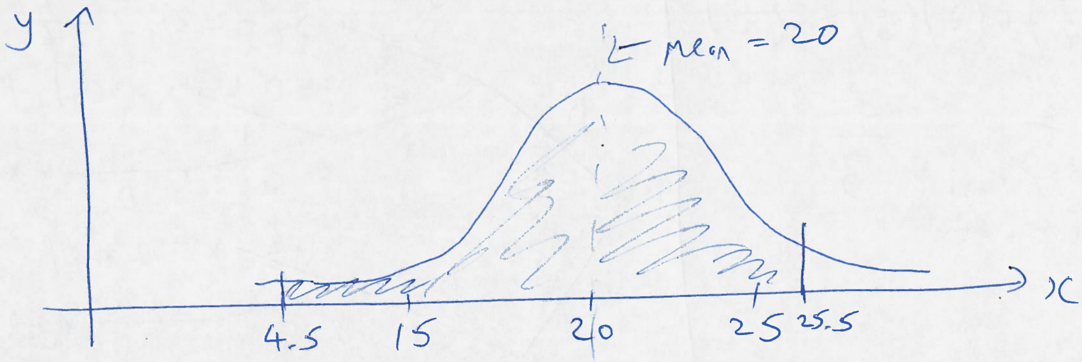
Binomial, $n = 200$, $p = .1$, $q = .9$

$P(X=3) = C(200,3)(.1)^3(.9)^{197} = .000001\ldots$

(b) Sketch the normal curve that best approximates the probability distribution of the random variable X. Be sure to include labelled axes, and a scale. Also, say on your sketch what is the mean and standard deviation of the normal curve that you are drawing.

$\mu = np = 20$

$\sigma = \sqrt{npq} = 4.24$



(c) Use the normal curve from part (b) to estimate the probability that the machine produced between 5 and 25 defective screws, inclusive.

Probability that Binomial is between 5 and 25, inclusive, best approximated by probability that normal is between 4.5 and 25.5

From Calculator, this is $\approx .9026$

14. (10 pts.) A box has four batteries, two of which are defective. I select batteries one at a time (without replacing them in the box), and test them. Each test takes half a minute. I keep going until I have found (and tested) *both* defective batteries, then I stop. Let X be the number of minutes this process lasts.

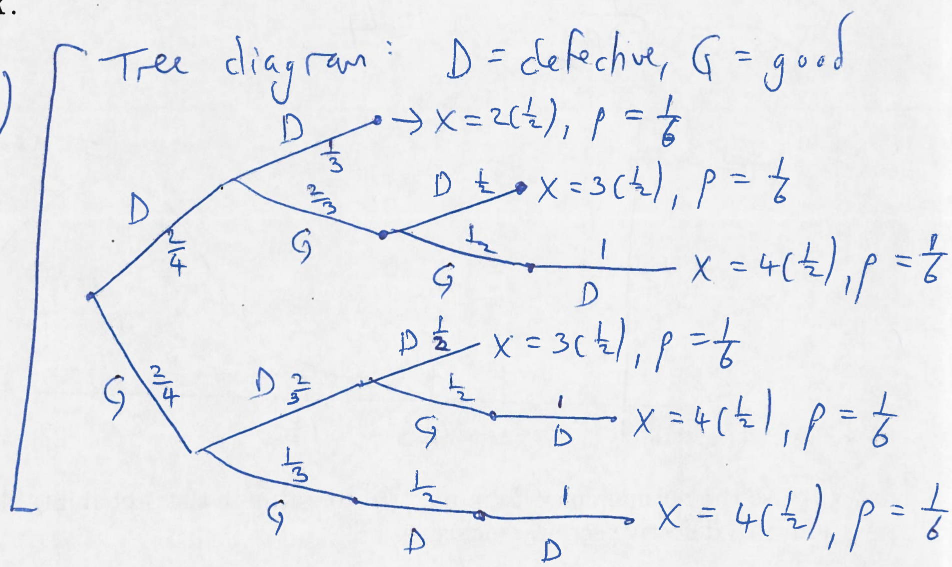
(a) Write down a probability distribution table for this experiment. (A tree diagram might help).

X (Number of minutes)	Probability of X
1	$\frac{1}{6}$
$1\frac{1}{2}$	$\frac{1}{3}$
2	$\frac{1}{2}$

(b) Calculate the expected value of X.

$$E(X) = 1\left(\tfrac{1}{6}\right) + \left(\tfrac{3}{2}\right)\left(\tfrac{1}{3}\right) + 2\left(\tfrac{1}{2}\right)$$

$$= 1\tfrac{2}{3} = \frac{5}{3}$$



(c) Calculate the variance of X.

$$E\left((X - E(X))^2\right) = \left(\tfrac{5}{3} - 1\right)^2\left(\tfrac{1}{6}\right) + \left(\tfrac{5}{3} - \tfrac{3}{2}\right)^2\left(\tfrac{1}{3}\right) + \left(\tfrac{5}{3} - 2\right)^2\left(\tfrac{1}{2}\right) = \frac{15}{108}$$

15. (10 pts.) A soft-drink company can produce up to 5 000 bottles of soda a day, in either regular or diet variety, and has an operating budget of up to \$5 400 per day. It costs \$1 to produce each bottle of regular soda and \$1.20 to produce each bottle of diet. There is 15 cents profit per bottle of regular soda and 17 cents profit per bottle of diet soda. The company knows that it can sell anything that it produces on a given day, and wishes to maximize daily profit.

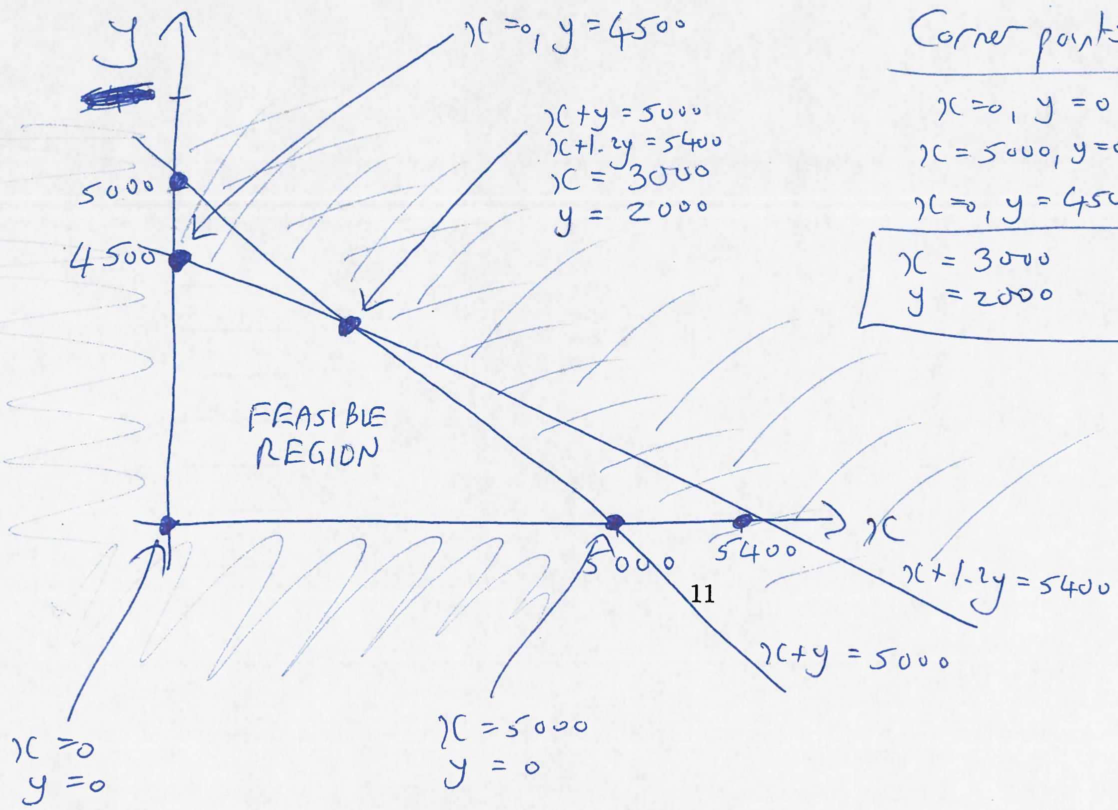
(a) Set this problem up as a linear programming problem. Be sure to say what the variables represent.

x = # bottles of regular soda produced
y = # " diet " "

Maximize $15x + 17y$ subject to

$$x + y \le 5000$$
$$x + 1.2y \le 5400$$
$$x \ge 0,\ y \ge 0$$

(b) By drawing a graph of the feasible region, solve the linear programming problem, and calculate what the maximum possible profit is.



Corner points	objective
$x=0,\ y=0$	0
$x=5000,\ y=0$	\$750
$x=0,\ y=4500$	\$765
$\boxed{x=3000,\ y=2000}$	$\boxed{\$810}$

Largest corner point, so this is maximum profit.